Delve AP Calculus Homework 3

For 1-9, $f(x)$ is everywhere continuous and differentiable. $f(2) = 2$, and $f(7) = 7$. Of the following, state whether the sentence is definitely false, definitely true, or could be either true or false with the information given.

1. $f(x)$ contains a hole discontinuity.
2. $f(x)$ contains a step discontinuity.
3. $f(5) = 0$.
4. $f'(5) = 0$.
5. $f(5) = 5$.
6. $f(n) = 5$ for some positive number n.
7. $f'(n) = 5$ for some positive number n greater than 2.
8. $f''(n) = 5$ for some positive number n greater than 2.
9. $f'(n) = 1$ for some positive number n.
10. Evaluate $\lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$. This expression finds the derivative for which function of x?

For 11-14, given $f(x)$, find $f'(x)$.

11. $f(x) = x^2 - 1$
12. $f(x) = (\sin x)(\ln x)$
13. $f(x) = e^{e^{e^{e^x}}}$
14. $f(x) = \frac{\sin x}{\tan x}$

For 15-17, given $f(x)$, find $f'(3)$.

15. $f(x) = x^2 - 1$
16. $f(x) = 2^x$
17. $f(x) = x^3 - 3x^2 + 3x$
18. A car's position from time t=0 to time t=12 can be modeled by $p_1(t) = \ln t$. According to this model, what speed was the car traveling at time t=2?
19. A different car's position from time t=0 to time t=12 can be modeled by $p_2(t) = \sin t$. According to this model, what is the difference between the speed the car was traveling at at time $t = \pi$ and the speed the car was traveling at at time $t = 2\pi$?
20. Graph an example of a function that is continuous between $x = -2$ and $x = 2$ but not differentiable at some point between $x = -2$ and $x = 2$.