Symmetry and the Complex Plane Syllabus

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About the Class

The aim of this class is to build the math we need to study a family of special spiral functions and their interactions. These interactions produce marvelous fractal pictures with layers upon layers of complexity and symmetry. We will be studying a lot about functions and complex numbers, and through this, come to understand how these fractals are generated. Much of this is based off the text *Indra's Pearls* by David Mumford, Caroline Series, and David Wright.

1 The Language of Symmetry

First, some pretty pictures and a discussion of why we are studying this and where the class will be going. Some prehistory about Klein and Bolyai, the definition of geometry, and a discussion about symmetry in overview, what kinds of symmetry exist, and how we can represent this symmetry. (Translations, rotations, reflections, the basic functions of symmetry.) Then we'll dive into the algebra of symmetry, including functions and groups, and how to conjugate functions.

2 A Delightful Fiction

An introduction to complex numbers and the complex plane, including some history. Real and imaginary parts, complex conjugates, and how to do arithmetic with complex numbers. Then we'll get into different ways of representing complex numbers, namely, $re^{i\theta}$, vectors, $cis(\theta)$, and $a + bi$, and why each of these forms is useful. Perhaps a small digression about how different representations can be used to prove trigonometric identities, if time.

3 Simple Complex Functions

Combining our first two classes, we'll talk about simple complex functions (sort of an oxymoron...) Functions of the form $S(x) = ax$, $T(x) = x + b$, or $I(x) = 1/x$ and their properties. We'll introduce the concept of the Riemann Sphere and the point at infinity, and talk about movements of the Riemann Sphere. Mostly focus on circle inversions: conjugate the inversion formula for a circle of any radius centered anywhere, talk about circles on the Riemann Sphere and flipping the Riemann Sphere/switching the radius, and where different points move to. We will talk about all of this in the context of the different representations of complex numbers that we discussed.

4 Many Marvelous Mobius Maps

A quick bit about spirals and conjugating spirals to different sinks and sources. Some more discussion of the Riemann Sphere as a review, before talking about the main topic of Mobius transformations. Some background math in matrices, then a discussion of the matrix representation and determinants. An introduction to the group $SL(2, \mathbb{C}$. Inverse functions and group dynamics. If possible we will talk about the symmetry of these transformations. More about circles and pairing functions, if possible.

5 The Schottky Dance

Circle pairings! This will be the conclusion to all the math we have learned, and we will finally understand how the fractal pictures are generated using circle pairings. We will talk about what makes a circle pairing, how they work, and what kind of symmetry they have.