Vectors and Beyond: Syllabus

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1 Course Description

So perhaps you've heard about vectors—you know, these things that have a magnitude and direction and stuff.

Of course, that understates how awesome they are. We'll take a look into the world of vectors, starting from what it means for something to be a vector. We'll introduce matrices as descriptions of linear transformations—things that turn vectors into other vectors in certain predictable ways. (Oh hey, matrices are useful after all!) As we explore the uses of vectors and matrices, we'll try to answer the questions that arise, such as: Why do we need a right-hand rule for cross products? What can we say about things that turn matrices into other matrices? And what happens to a coconut when you stretch it?

Planned topics include: vector spaces, matrices and matrix groups, dot and cross products, covariance and contravariance, tensors, and applications to physics.

2 Prerequisites

Familiarity (or at least comfort) with mathematical reasoning and proofs. Most likely this implies you'll also be proficient in algebra and geometry at the high school level. It will really help to have seen polynomials, complex numbers, and the trigonometric functions.

3 Class Format

Class time will be split between lecture and problem solving, with a break in the middle. There will be optional homework (a few problems per week).

4 Approximate Schedule of Topics

Day 1 — Vectors

- Playing with vectors in $\mathbb{R}^3$
- Vector addition, dot products, and cross products
- Defining vector spaces

Day 2 — Matrices

- Matrix multiplication
- Matrices as linear transformations
- Introduction to matrix groups
- Quaternions and complex numbers as 2×2 matrices

Day 3 — More about matrices

- Matrix groups continued
- Changing basis
- Eigenvalues and eigenvectors
- Hyperbolic rotations and Special Relativity

Day 4 — Dot products revisited

- Dot products and basis changes
- Generalizing to bilinear forms
- Covectors, covariance, contravariance, and coconuts
- More Special Relativity

Day 5 — Cross products revisited

- Algebras (yes, plural)
- Cross products and wedge products
- Bivectors and pseudovectors
- Rotational motion

Day 6 — Introduction to tensors

- Tensors as generalizations of matrices
- Playing with tensors
- The Levi-Civita symbol
- Linear elasticity

Day 7 — More about tensors

- The tensor product
- Tensor spaces
- Tensors and basis changes
- Covariance and contravariance revisited

Day 8 — Bonus material, if time allows

- More about wedge products
- Modules
- Electricity and magnetism (requires calculus)