MIT HSSP 2009

M2048 **SYLLABUS** Spring 2009

**Course**: Topics in Mathematics **Instructor**: Andrew Spieker

**Room**: 5-217 **Class** **Time**: 10:00 AM – 12:00 PM

**E-mail**: spieker.a@neu.edu **Course Website**: https://esp.mit.edu/learn/HSSP/2009\_Spring/Classes/M2048/index.html

**Course Description and Topics**: This takes into account the feedback you have given me. I took it very seriously; I tried not to leave ANYTHING out. There are two important points that I want to make: If you’re worried that this will be too difficult, don’t be. Most of it is just fancy words, and you’re not being tested on it. You won’t be expected to memorize anything. Second of all, if you’re thinking that you’ve seen most of this stuff before, I promise you that I will be putting a different spin on it. You will get something out of this! I will post a summary of things we did in class each week, so you can refer back to it if you wish.

**Week 2: Topology; Short Derivation of the Quadratic formula**

We will discuss the fundamental notions of topology, give applications to theorems in algebra (with polynomials). We will discuss the concept of homeomorphism, and when two topological spaces are homeomorphic. We apply this to the standard English Alphabet, to see what letters can be “squished” into each other.  
  
We then will move into orientability of manifolds, and we will construct our own Mobius Strips and perform a couple of experiments with them. We will talk about the orientability of other spaces and give possible applications.  
  
As promised, I will derive the quadratic formula as time permits.

**Week 3: Short Discussion on Real Numbers; Limits, Continuity, Derivatives**

We will keep the discussion on the real numbers very short: 15-20 minutes maximum. From this, we will show that limits exist, and that the types of functions we usually deal with are continuous everywhere they’re defined.  
  
We will then discuss the notion of derivatives as a limit, and as a function. We will characterize differentiability in terms of local linearity. As time permits, we will give generalizations of calculus in n dimensions, and attempt to try to describe what 4-dimensional space “looks like,” and discuss partial derivatives.

**Week 4: Finish Up Derivatives, Introduction to Integral Calculus**

Topics that were unfinished from the previous week will be finished here. We will then derive some common derivative formulas.  
  
All of a sudden we switch to the “seemingly” unrelated topic of continuous change, and apply it to area under a curve. We construct an approximation of Riemann Sums, and take a limit as the approximation becomes better and better. As time permits, we will look at Lebesgue Integration, and try to understand the concepts of continuous change.

**Week 5: Finish Up Integral Calculus, Discussion on Set Theory, and Introduction to Groups**

Topics remaining unfinished from the previous week will be finished here. We will then prove the fundamental theorem of calculus, linking derivatives and integrals together.

We will abruptly stop our discussion of calculus, and discuss some of the interesting notions of “set theory.” Possible topics include Peano’s Axioms, contradictions of “the set of all sets,” point-set theory, and topology of the real number line.

We will discuss sets that are equipped with a binary operation that forms a group; an algebraic structure very important to algebra, geometry, and number theory.

**Week 6: Short Discussion of Trigonometry; Vectors, Dot/Cross Product, Physics Applications**

First, we will discuss some basic trigonometry, at least as pertinent to things we will do later in the course.  
  
We will then introduce vectors as geometric quantities and as abstract quantities, providing both concept AND definition. We will define the dot product and cross product of vectors, and prove the Pythagorean Theorem using the idea of dot product. As time permits, we will apply these things to physical quantities (they will all be defined,) and talk about vector spaces, linear transformations, eigenvectors, and dimension of the image and nullspace of linear transformations.

**Week 7: Contour Integration/Vector Fields; Complex Variables, Conformal Mappings, Discussion on Non-Euclidean Geometry**

We will cover contour integration (sometimes called line integration) in three dimensions, and apply it to vector fields, and interpret the results geometrically. We will then switch over to complex variables (variables of the form z=a+bi) and discuss functions that map open subsets of the complex plane into other open subsets of the complex plane. Of interest are conformal mappings; that is, nontrivial mappings that preserve angles between the contours of the maps.  
  
We will then have a discussion of Non-Euclidean Geometry; that is, the geometry not of lines, but of ellipses and hyperbolas. This will tie in nicely.

**Week 8: More on Groups; Rings, Fields, and Algebraic Structures; Possibly Category Theory(?); Discussion on Induction**

This is the second in the series of abstract algebra lectures. We will discuss more on groups, talk about rings, and discuss fields and connect them to things you already know about polynomials.  
  
As time permits, we will talk about category theory, and possibly discuss other things like induction.

**Brain Teasers:** Also, for the people who wanted more probability, you’ll be happy to know that I’ve decided to make the weekly brain teasers based on probability theory and expectation. I will talk about them at the end of class each week, and post the solutions sometime on Tuesday or Wednesday, after you’ve had time to think about them for a little while.

**Homework:** There is no formal homework for this course. You already have homework from school, and I have homework from my classes; there’s no need ☺. However, if you want me to provide you with material or good sources, I will be more than happy to. Periodically, I will post problems, but you should feel in no way *obligated* to do them for a grade or some type of ‘judgment’ on my part. This course is based largely on exploration of different topics, so I care more about you thinking in new ways as opposed to working to understand every little detail we cover.

**Class Structure**: A good portion of this course will be devoted to demonstrations and examples. At the beginning of each session, we will start with a review of the previous session, possibly enjoy some munchkins/donut holes or whatever snacks you may enjoy. Then, we will break the session down into mini-lectures and activities. I will try to *never* lecture for more than fifteen to twenty minutes at a time. NOTHING is more annoying than just having to sit and listen to someone talk for two hours. I won’t put you through that. Expect to think in new ways, about new topics!

**Final Words**: The main objective of the course is for you to enjoy yourself! Have a good time with this; it will allow you to have a deeper appreciation of what we cover.

**Attendance:** Of course, we’re all busy, and things come up! If you miss a class, don’t stress out about it, but please try your best to be here for every class though, and on time (class will typically start at about 10:05); you are all what makes the class function. If you know you are going to be absent, please e-mail me in advance just so I know how many copies to make.